LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **NOVEMBER 2024**

UMT 2501 – ANALYTICAL GEOMETRY

Date: 09-11-2024 Dept. No. Max. : 100 Marks

Time: 09:00 am-12:00 pm

	SECTION A - K1 (CO1)	
	Answer ALL the Questions -	**(10 x 1 = 10)**
1.	**Answer the following**	
a)	If the polar of P with respect to a parabola passes through Q, point out that polar of Q passes through P.	
b)	Define rectangular hyperbola.	
c)	Define skew lines.	
d)	Find the equation of the sphere with centre (-1, 2, -3) and radius 3 units.	
e)	Define a cone.	
2.	**Fill in the blanks**	
a)	If e is the eccentricity of the conic, where e < 1, then the conic generated is _____.	
b)	The hyperbola which has the asymptotes $lx+my+n=0$ and $l_1x+m_1y+n_1=0$ is _____.	
c)	The direction cosines of the Y-axis is _____.	
d)	If the plane passes through the centre of the sphere, the intersection of plane and sphere is called a _____.	
e)	The fixed straight line in a right circular cone is called _____ of the cone.	
	SECTION A - K2 (CO1)	
	Answer ALL the Questions	**(10 x 1 = 10)**
3.	**MCQ**	
a)	If the pole of parabola is at the focus, say (a, 0), then the polar equation of the pole is (i) $x+a=0$ (ii) $x-a=0$ (iii) $y+a=0$ (iv) $y-a=0$	
b)	The distance between the points (r_1,θ_1) and (r_2,θ_2) is (i) $r_1^2+r_2^2+2r_1r_2\cos(\theta_1-\theta_2)$ (ii) $r_1^2+r_2^2-2r_1r_2\cos(\theta_1-\theta_2)$ (iii) $r_1^2+r_2^2+2r_1r_2\cos(\theta_1)$ (iv) $r_1^2-r_2^2-2r_1r_2\cos(\theta_2)$	
c)	Let (l_1,m_1,n_1) and $(l¿¿2,m_2,n_2)¿$ be the direction cosines of two lines and θ be the angle between them. Then the equation $l_1l_2+m_1m_2+n_1n_2=0$ is the condition for the lines to be (i) parallel (ii) perpendicular (iii) non-intersecting (iv) intersecting	
d)	The center of the sphere $x^2+y^2+z^2+2x-4y+6z+5=0$ is (i) $(-1,2,-3)$ (ii) $(-1,-2,3)$ (iii) $(1,-2,-3)$ (iv) $(1,2,3)$	
e)	Any plane which intersects the cylinder whose equation is of _____ degree is a conic. (i) First (ii) Second (iii) Third (iv) Fourth	
4.	**True or False**	
a)	If p be the perpendicular on the tangent at P from the center of an ellipse, then $p\cdot CD=ab$.	
b)	The equation of the asymptotes does not differ from that of the hyperbola only in the constant term.	
c)	Two straight non-intersecting lines in space are called non-skew lines.	

d)	The plane passing through the centre of the sphere is called a circle.
e)	A surface generated by a straight line which passes through a fixed point and makes a constant angle with a fixed straight line through the fixed point is a cylinder.

SECTION B - K3 (CO2)	
Answer any TWO of the following	**(2 x 10 = 20)**
5.	Derive the equation of pair of tangents to the parabola $y^2=4ax$ from the point (x_1, y_1).
6.	Find the asymptotes of the hyperbola $3x^2-5xy-2y^2+17x+y+14=0$.
7.	Discover the equation of the plane passing through the points (2, 2, 1), (2, 3, 2) and (-1, 3, 1).
8.	Examine the equation of the cone with vertex O and base curve, the conic in which the surface $ax^2+by^2+cz^2=1$ is cut by the plane $l_1x+m_1y+n_1z=p$.
SECTION C – K4 (CO3)	
Answer any TWO of the following	**(2 x 10 = 20)**
9.	Examine the tangent to a rectangular hyperbola terminated by its asymptotes, is bisected at the point of contact and encloses triangle of constant area.
10.	Derive the equation for a rectangular hyperbola, referring to its asymptotes as axes.
11.	Show that the lines $\frac{x+1}{-3}=\frac{y+10}{8}=\frac{z-1}{2}$ and $\frac{x+3}{-4}=\frac{y+1}{7}=\frac{z-4}{1}$ are coplanar. Find also the point of intersection and the plane through them.
12.	Find the equation of sphere which passes through the circle $x^2+y^2+z^2-2x-4y=0, x+2y+3z=8$ and touches the plane $4x+3y=25$.
SECTION D – K5 (CO4)	
Answer any ONE of the following	**(1 x 20 = 20)**
13.	(a) Define conjugate diameter of an ellipse and examine whether the product of the focal distance of a point on an ellipse is equal to the square of the semi-diameter which is conjugate to the diameter through the point. (b) Investigate this statement: The locus of midpoints of chords of a parabola which subtend a right angle at the vertex is another parabola whose latus rectum is half the latus rectum of the parabola. (10 + 10)
14.	(a) Analyze and write down the equation of orthogonal projection of the line $\frac{x-2}{4}=\frac{y-1}{2}=\frac{z-4}{3}$ on to the plane $8x+2y+9z-1=0$. (b) Determine the equation of the enveloping cylinder of the surface $ax^2+by^2+cz^2=1$ having the generator parallel to $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$. (10 + 10)
SECTION E – K6 (CO5)	
Answer any ONE of the following	**(1 x 20 = 20)**
15.	(a) Trace the curve $\frac{10}{r}=3\cos\theta+4\sin\theta+5$. (10 + 10) (b) Compute the symmetrical form of the line $3x-2y+z-1=0=5x+4y-6z-2$.
16.	(a) Find the equation of the sphere through the four points (2, 3, 1), (5, -1, 2), (4, 3, -1) & (2, 5, 3). (b) Find the bisector of the acute angle between the planes $x+2y+2z-3=0$ and $3x+4y+12z+1=0$. (10 + 10)

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